An Analytical Model for Point Source Pollutants in an Urban Area with Mesoscale and Removal Mechanisms

R. Latha[1,2,3], K. Lakshminarayanachari[2,3], C. Bhaskar[2,3]

[1]Department of Mathematics, Jyothy Institute of Technology, CIIRC, Bengaluru, Karnataka, India, [2]Department of Mathematics, Visvesvaraya Technological University, Belagavi, Karnataka, India, [3]Department of Mathematics, Sai Vidya Institute of Technology, Bengaluru, Karnataka, India

Abstract

Aim: Urbanization, rapid industrialization, and related anthropogenic activities are the main reasons for air pollutant emissions and poor air quality. Continued deterioration of air quality in urban areas requires effective measures to control air pollution. The purpose of developing a model is for a better understanding the physical, chemical, and dynamic properties of air pollution and meteorology. **Materials and Methods:** In this work, an analytical approach has been used to analyze the diffusion of air pollutants emitted from point sources is presented. This article reports the method of solving three-dimensional atmospheric diffusion equation using the Fourier transform technique, variable separable method, and series solution to obtain analytical solution. **Results:** The present work focuses on the effect of dry deposition and wet deposition on the concentration of primary impurities. The concentration of primary impurities decreases when the removal rate of dry and wet deposition increases with respect to height and distance. **Conclusion:** The focus of this research article is to learn the effects of removal mechanisms such as dry and wet deposition of primary pollutants. It is found that the contaminant concentrations are close to point sources and gradually decrease toward nonsource areas due to diffusion and removal mechanisms in an urban area.

Keywords: Advection–diffusion equation, atmospheric impurities, dry deposition, urban heat island, wet deposition

Introduction

Air pollution is considered to be the most important environmental risk factor for health. The smoky air that surrounds cities often contains dangerously high levels of particulate matter are called PM2.5. These pollutants are associated with lung and heart disease and are known to affect cognitive function and the immune system. Air pollution also harms our natural environment. It reduces the oxygen supply to our oceans, makes plant growth difficult, and contributes to climate change. Other harmful environmental impacts include soil and waterway depletion, endangered freshwater sources, and reduced crop yields. Hence, it is of great importance to study the dispersion of pollutants.

Wind speed measures the air pollutants mixed in relation to wind speed, and their direction leads to the widespread diffusion of pollutants. To develop a precise model or method to illustrate the manner, in which air pollution is transported and dispersed for a given locale, information about the pollutant source is needed. This information generally includes surrounding geographic features, quantity and types of pollutants emitted, effluent gas conditions, stack height, and influential meteorological factors.

The advection–diffusion equation is analytically solved for the steady state by making appropriate assumptions about the wind speed profile and the eddy diffusion coefficient. Griffiths[1] predicted knowledge of large winds is insufficient to predict urban air pollution. Sharan[2] formulated a mathematical model of air pollution propagation in weak winds, taking into account all directional propagation and advection along average winds. The analytical solution of the advection–diffusion equation is the basis for the explanation and understanding of the dispersion phenomenon. Pasquill and Smith[3] determined, it is expressed in a mathematically closed

Access this article online

Quick Response Code:

Website: www.ijehe.org

DOI: 10.4103/ijehe.ijehe_59_22

Address for correspondence: Dr. K. Lakshminarayanachari, Department of Mathematics, Sai Vidya Institute of Technology, Bengaluru - 560 064, Karnataka, India. Visvesvaraya Technological University, Belagavi, Karnataka, India. E-mail: lnc2research@gmail.com

How to cite this article: Latha R, Lakshminarayanachari K, Bhaskar C. An analytical model for point source pollutants in an urban area with mesoscale and removal mechanisms. Int J Env Health Eng 2023;12:21.

Received: 30-11-2022, **Accepted:** 15-03-2023, **Published:** 31-08-2023

form using all parameters, making it easy to investigate the effect of individual parameters on pollutant concentrations. Dilley and Yen[4] investigated the effects of mesoscale wind on the pollutant distribution of infinite sources. Arora *et al*.[5] presented an analytical approach for investigating the delayed removal process of air pollutants from line sources using variable wind profiles and diffusion coefficients. Ermak.[6] obtained the analytical solution of the atmospheric dispersion equation, assuming that the eddy diffusion coefficient is constant as a function of the downdraft distance and the average wind speed. A generalized mathematical model explaining the crosswind-integrated concentration is presented for the spread of pollutants released from continuous sources of the atmospheric boundary layer with surface deposition given by Pramod and Maithili.[7]

The solution for air-pollutant enrichment is obtained through an integrated and diverse embedding process that takes into account chemical reaction processes by Naresh and Nath.[8] The effects of mesoscale wind and chemical reactions about pollutants in the presence and absence of mesoscale wind were investigated by Joseph and Agarwal.[9] The solution of the two-dimensional convection–diffusion equation using the Laplace transform method is formulated by Buske.[10] They also discussed solving the three-dimensional (3D) convection–diffusion equation for wind speed profiles. Wortmann *et al*.[11] proposed a solution to the convection–diffusion equation by adopting a generalized integral transform method. Pal and Khan[12] reported a time-dependent model of air pollutants from point sources and concluded that pollutant concentrations decrease as pollutants escape from the top of the inversion layer. The analytical approach was developed by Khan *et al*.[13] to analyze the removal mechanism of nonreactive air pollutants for various complex air conditions. Narayanachari KL *et.al*.,[14] used a numerical model to investigate the causes of mesoscale airflow in the distribution of air pollutants. Lakshminarayanachari *et al*.[15] have developed a two-dimensional numerical model for studying the diffusion of primary pollutants in residential areas with chemical reactions and dry deposition. Many researchers have reported the occurrence of health problems due to air pollution. Pope[16] found that air pollution causes chronic bronchitis, leading to increased mortality. However, the work reported so far does not integrate the effects of mesoscale airflow on point sources.

The study by Lakshminarayanachari *et al*.[17] elucidates that the mesoscale wind prevents the diffusion of pollutants that would increase the concentration of pollution in the atmosphere and also studied that urban heat island (UHI) increases the concentration of pollutants and helps to move vertically upward, resulting in pollution becoming more serious. Verma *et al*.[18] have presented analytical model of the diffusion of air impurities with variable wind velocity and constant removal rate. The solutions of advection–diffusion equation were obtained by analytical approach, with the assumption that eddy diffusivities to be linear functions of downwind distance with physically relevant boundary conditions. They reported this under the assumption that pollutants would be transported horizontally by large winds and the mesoscale winds generated would transport them both vertically and horizontally. However, these works did not compact with the consequences of mesoscale air current for the point source.

This paper sought to develop an analytical method for studying the spread of air pollutants emitted from elevated reference points integrated into the mesoscale airflow generated by the UHI. This task uses the convection–diffusion equation to analyze the diffusion of air pollutants in the presence of large winds, eddy diffusion, and thermal diffusion. In this article, we study the effect of removal mechanisms such as dry deposition and wet deposition on the concentration of pollutants along vertical height and horizontal distance.

Materials and Methods

Mathematical model formulation

Based on the gradient transport theory, the diffusion equation for describing the dispersion of atmospheric pollutants in the atmosphere is given by Equation 1:

$$\frac{\partial c}{\partial t}+u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}+w\frac{\partial c}{\partial z}=\frac{\partial}{\partial x}\left(k_x\frac{\partial c}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y\frac{\partial c}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_z\frac{\partial c}{\partial z}\right)-\alpha C \tag{1}$$

Where C represents the concentration pollutants in the atmosphere. *u, v,* and *w* are the wind speed and k_x, k_y and k_z are diffusivities in x, y, and z direction. α represents the removal rate of atmospheric pollutants naturally.

In the current problem, the point source is kept at a height of h_s meters from ground level. Here, it assumed that pollutants are discharged from a point source at a constant rate and the pollutants move in a horizontal direction parallel to the large-scale airflow. The point source is being fixed at the origin with coordinates x=0, y=0. The diffusion of pollutants is determined at a downwind length l = 5 km.

To develop this mathematical model, the following assumption has been made:

I. The release of atmospheric pollutants point source is steady rate
II. The condition for a steady state is considered to be, $\frac{\partial c}{\partial t}=0$
III. The direction of x coordinate is aligned with the average wind speed (u = U and v = 0)
IV. The horizontal advection of the airflow dominates the parallel diffusion.

$$\text{i.e. } u\frac{\partial c}{\partial x}\gg\frac{\partial}{\partial x}\left(k_x\frac{\partial c}{\partial x}\right) \tag{i}$$

Considering the above assumptions, the Equation 1 becomes as Equation 2.

$$u(x)\frac{\partial c}{\partial x}+w(z)\frac{\partial c}{\partial x}=k_y\frac{\partial^2 c}{\partial y^2}+k_z\frac{\partial^2 c}{\partial z^2}-\alpha C \quad (2)$$

Where x, y, and z are the coordinate axes, U (x) is the speed of the airflow in the x direction which found to vary with vertical distance from the ground level. It can be written in the following form Dilley and Yen.[4]

$$U(x)=U_0(1-ax) \quad (ii)$$

Where U_0 represents the average speed of the wind.

W (z) = $U_0(az)$ indicates air current velocity along the direction of z.

Usually, $K_y>K_z$ in the atmospheric conditions.

The boundary constraints in Equation 2 are:

(I) Impurities are discharged from the elevated point source with the concentration of Q positioned at a coordinate $(0,0,h_s)$

$$C(x,y,z)=\frac{Q\delta(y)\delta(z-h_s)}{U(x)} \text{ at } x=0,\ 0\le h_s\le H \quad (3)$$

Where, h_s-stack height, δ-Dirac delta function, and H-mixing height

(II) Concentration of impurities becomes zero as it tends to be away from the point source in y direction, i.e.,

C (x, y, z) =0 when y → ± ∞ (4)

(III) The impurities are reflected at height h from the ground surface i.e.,

$$\frac{\partial C}{\partial z}=v_d C \text{ at z = h} \quad (5)$$

Where v_d is the dry deposition.

(IV) There is some diffusion flux at the vertical height H from the ground surface

$$k_z\frac{\partial c}{\partial z}=w_s C \text{ at } z=H \quad (6)$$

Where w_s is the wet deposition.

Recent studies show that large-scale wind speeds are considered constant. That is, u = U_0 is presumed that the horizontal mesoscale wind fluctuates in the vertical direction equal to u. The vertical mesoscale air flow w (z) can be calculated by integrating the continuity equation.

$$w_e=aU_0 z \quad (iii)$$

Where *a* is the proportionality constant.

$$U(x)=u+u_e=U_0(1-ax) \quad (iv)$$

Method of solution

Then, by employing the following dimensionless parameter procedure, the partial differential equation (2), which describes the diffusion of air pollutants and boundary constraints, is reduced to dimensionless to arrive at the solution:

$$x^*=\frac{K_{z_0}x}{U_0H^2},y^*=\frac{y}{H},Z^*=\frac{z}{H},U^*=\frac{U}{U_0},C^*=\frac{U_0H^2C}{Q},$$

$$\beta^*=\frac{K_Y}{K_{z_0}},\gamma^*=\frac{K_z}{K_{z_0}} \quad (v)$$

$$\delta(y^*)=H\delta(y),\ \alpha^*=\frac{U_0H^2\alpha}{K_{Z_0}},k^*_w=\frac{H^2k_w}{K_{Z_0}} \quad (vi)$$

Where K_{Z_0} is the reference diffusivity and U_0 is the reference wind velocity. On dropping asterisks (*), the Equation 2 and the boundary conditions given by Equations 3–6 may be made in the dimensionless form as found below

$$(1-ax)\frac{\partial C}{\partial x}+az\frac{\partial C}{\partial Z}=\beta\frac{\partial^2 c}{\partial y^2}+\gamma\frac{\partial^2 c}{\partial z^2}-\alpha c \quad (7)$$

$$C(x,y,z)=\frac{Q\delta(y)\delta(z-h_s)}{U(x)} \text{ at x = 0} \quad (8)$$

$$C=0 \quad \text{when } y\to\pm\infty \quad (9)$$

$$\frac{\partial C}{\partial z}=NC \ at\ z=1 \quad (10)$$

$$\frac{\partial C}{\partial z}=MC\ at\ \text{z=h / H} \quad (11)$$

The Equation 7 is solved by employing Fourier transform technique using boundary conditions given by Equation 8 to Equation 11.

Taking Fourier transform of the Equation 7 with respect to "y" to get

$$\frac{(1-ax)\partial\bar{C}}{\partial x}+p^2\beta\bar{C}=\gamma\frac{\partial^2\bar{C}}{\partial z^2}-az\frac{\partial\bar{C}}{\partial Z} \quad (12)$$

Where $\bar{C}=\bar{C}(x,p,z)$ is the Fourier Transform of c with respect to y and p

By considering the Fourier transform, the boundary conditions become

$$\bar{C}=(x,y,z)=\frac{Q\delta(y)\delta(z-h_s)}{U(x)}\ at \text{ x=0} \quad (13)$$

$$\bar{C}=0 \text{ when y}\to\pm\infty \quad (14)$$

$$\frac{\partial\bar{C}}{\partial z}=N\bar{C}\ at\ z=1 \quad (15)$$

$$\partial\bar{C}/\partial z==M\bar{C}\ at\ \text{z = h / H} \quad (16)$$

In this method of separation of variables using the Equation 12 and the following trial solution:

$$\bar{c}=X(x)Z(z) \quad (17)$$

Where Z (*z*) is a function of z only and *X(x)* is a function of only x.

By substituting the Equation 17 in Equation 12, the ordinary differential equations are obtained

$$\frac{(1-ax)}{X}\frac{dX}{dx}+(p^2\beta+\lambda^2)=0 \quad (18)$$

$$\gamma\frac{d^2z}{dz^2}-az\frac{dz}{dz}+\lambda^2 Z=0 \quad (19)$$

The term λ^2 represents the separation constant

The solution of Equations 18 and 19 are of the form

$$X=C_1(1-ax)^{\frac{p^2\beta+\lambda^2}{\alpha}} \quad (20)$$

$$Z=a_0 f(z)+a_1 g(z) \quad (21)$$

Where a_0, a_1, and C_1 are arbitrary constants and

$$f(z)=1-\frac{\lambda^2}{2!\gamma}z^2-\frac{\lambda^2(2a-\lambda^2)}{4!\gamma^2}z^4-\frac{\lambda^2(2a-\lambda^2)(3a-\lambda^2)z^6}{6!\gamma^3}$$

$$g(z)=z+\frac{(a-\lambda^2)}{3!\gamma}z^3+\frac{(a-\lambda^2)(3a-\lambda^2)}{5!\gamma^2}z^5+\frac{(a-\lambda^2)(5a-\lambda^2)(3a-\lambda^2)z^7}{7!\gamma^3}$$

After substituting the values of X (x) and Z (z) from the Equations 20 and 21, the following equation is obtained

$$\bar{C}=(1-ax)^{\frac{(p^2\beta+\lambda^2)}{\alpha}}\left(a_0 f(z)+a_1 g(z)\right) \quad (22)$$

The term C_1 is considered to be 1 without loss of generality. After using the boundary conditions,

$$\frac{\partial C}{\partial z}=NC \ at \ z=1$$

$$\text{The value of } N=\frac{f(1)}{f'(1)} \quad (23)$$

$$\partial\bar{C}/\partial z=M\bar{C} \ at \ z=h/H$$

$$M=\frac{f(h/H)}{f'(h/H)} \quad (24)$$

Again using the boundary conditions, $\bar{C}(x,y,z)=\frac{\delta(z-h_s)}{(1-ax)}$ x = 0 and as well as applying

$\int_0^1 \delta(z-h_s)f_n(z)\,dz=f_{n(h_s)}$ and

$\int_0^1 z^p f_m b(z) f_n(z)\,dz=0$ m$\neq n$, the solution can be written as

$$\bar{C}=(1-ax)^{\frac{(p^2\beta+\lambda^2)}{\alpha}}\frac{f(h_s)}{p}f(z),\text{where } p=\int_0^1 f^2(z)\,dz \quad (25)$$

After taking the inverse Fourier transform of the Equation 25, it can be written as

$$C=0.28209\sqrt{\frac{a}{\beta\log\left(\frac{1}{1-ax}\right)}}(1-ax)^{(\lambda^2)/\alpha}\frac{f(h_s)}{p}f(z)\ exp\left(\frac{ay^2}{4\beta\log(1-ax)}\right) \quad (26)$$

Results

The results of this research work are presented in Figures 1-6 to analyze the diffusion of air pollutants in the civic area.

Figure 1 depicts the variation of pollutant concentration with downwind distance for various values of dry deposition. It can be observed from the graph that, as distance (x) increases, the concentration of pollutant decreases. It can also be noticed that the concentration is more near the point source at origin then it is gradually decreasing after the point source and reaching zero.

Figure 2 reveals the variation of pollutant concentration with downwind distance for various values of wet deposition. It can be observed from the graph that, as distance (x) increases, the concentration of pollutant decreases.

Figure 3 gives the variation of pollutant concentration profile with height for various values of dry deposition values. From the figures, it is observed that the concentration is decreasing when height is increasing. Furthermore, it is observed that the concentration of impurities is high near the point source at origin, then it is gradually decreasing. Further, it is found that the concentration of pollutants is decreasing when dry deposition velocity increases.

Figure 4 indicates the variation of pollutant concentration profile with height for various values of wet deposition values. From the figure, it is observed that the concentration is decreasing when height is increasing. Furthermore, the pollutant concentration is high near the point source at origin, then it is gradually decreasing. Further, the concentration of pollutants is decreasing when wet deposition increases.

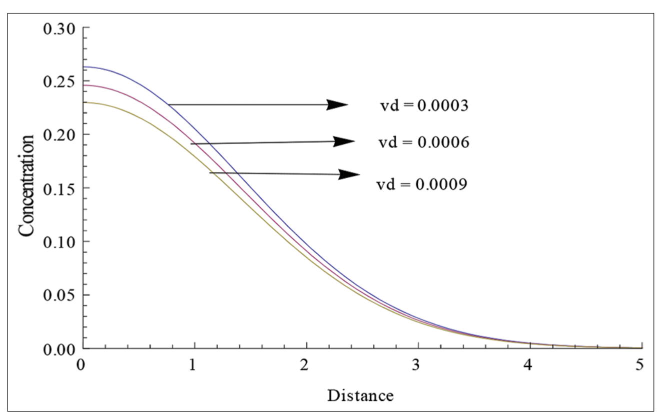

Figure 1: Concentration versus distance of primary pollutants for various values of dry deposition

Figures 5 and 6 exhibit the result of the concentration profile against the vertical height for various values of downward distance for different values of dry deposition values. Here, it is observed that, as vertical height increases, the concentration of pollutant is decreasing when $v_d = 0.0001$ and $v_d = 0.0004$. It is also observed that the concentration of pollutant is also decreasing when the downwind distance increases for both values of $v_d = 0.0001$ and $v_d = 0.0004$.

Figure 6 shows the variation of concentration with vertical height for different distances when $vd = 0.0004$.

Discussion

The UHI consequence produces its own mesoscale winds, and accordingly, it prevents the diffusion of the pollutants which will result in enhancing the pollution in an environment. The UHIs add to the development of haze of contaminated pollutants and also help these pollutants to circulate in an upward direction, thus making the problem of pollution more severe. It should be understood that the reason for the conversion of large cities into "UHIs" is attributed to anthropogenic aspects. Hence, combined efforts should be made in the task of UHI and for the creation of cooler and healthier cities.

The developed analytical model for studying the effects of primary pollutant concentrations in the horizontal and vertical directions discharged from an elevated point of reference at the source of the city was examined. Appropriate boundary conditions are given to explain the point source. Contaminants are expected to undergo removal mechanisms such as dry and wet deposits. The problem was solved using the Fourier transform, variable separable method, and series solutions.

An analytical solution for the computation of air concentration discharged from the point source undergoing dry and wet deposition process is presented. The results of this article are presented in Figures 1–6 to analyze the diffusion of air pollutants in the civic area. The concentration of primary pollutants decreases as removal mechanisms dry and wet deposition rate increases. The results of the present work are compared with Verma *et al*.[18] and Nirmaladevi *et al*.,[19] and good agreement was found.

The results of the current research findings can be used to increase the level of credibility in complex model predictions and to identify variables such as wind velocity and atmospheric solidity, which should be investigated more closely by complex modeling studies.

Conclusion

Analysis of the concentration of pollutants released from point sources in the presence of wet and dry deposits in the

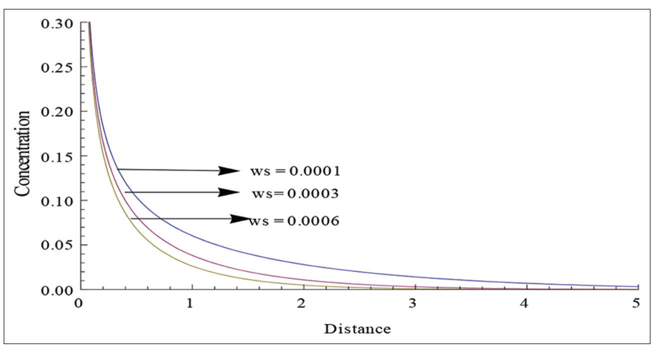

Figure 2: Concentration versus crosswind distance of primary pollutants for various values of wet deposition

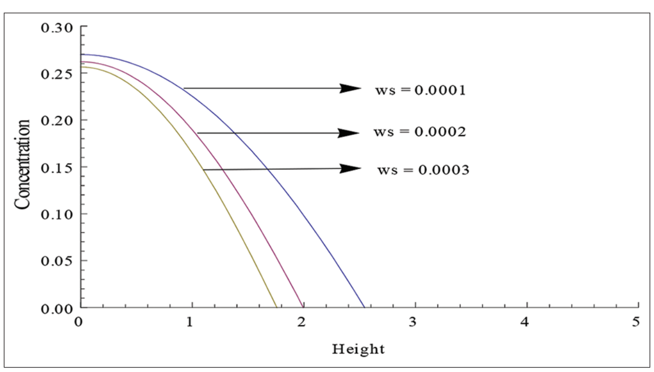

Figure 4: Concentration versus height of primary pollutants for various values of wet deposition

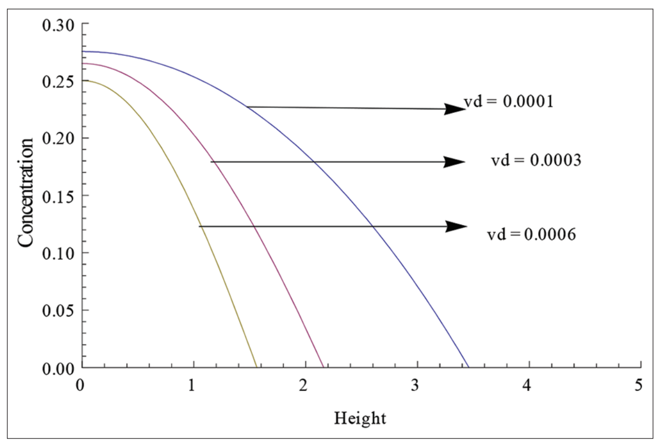

Figure 3: Concentration versus height of primary pollutants for different values of dry deposition

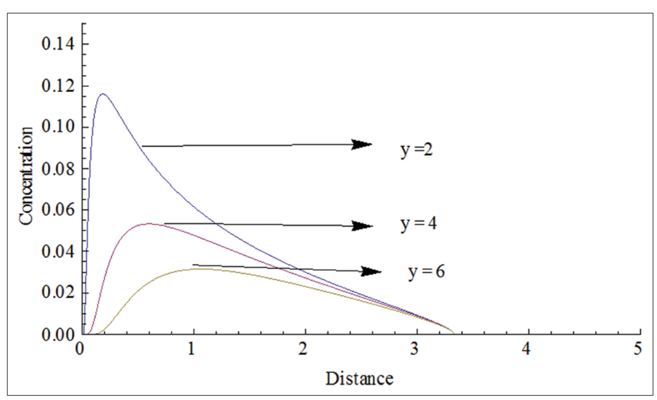

Figure 5: Variation of Concentration with distance for different values of crosswind distances when vd=0.0001

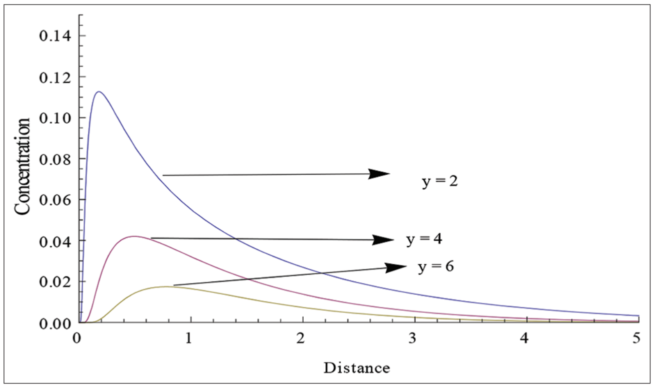

Figure 6: Variation of cConcentration with vertical height for different distances when vd=0.0004

city is presented using a 3D analytical model. The focus of this research article is to learn the effects of the removal mechanism of dry and wet deposition of primary pollutants. The results were analyzed for pollutants primarily in terms of horizontal distance and vertical height. It is observed that, as the proportion of removal mechanisms increases, the concentration of pollutants with respect to the horizontal distance and vertical height of the urban area decreases. Contaminant concentrations are close to point sources and gradually decrease toward nonsource areas due to removal mechanisms and diffusion.

Acknowledgments

The researchers wish to extend their sincere gratitude and appreciation toward the support from the Management and Principal of Sai Vidya Institute of Technology, Bengaluru. The researchers also wish to extend their sincere gratitude and appreciation toward the support from the Management and Principal of Jyothy Institute of Technology, Bengaluru.

Ethics code

This paper has never previously been published. This work was drafted by all of the authors.

Financial support and sponsorship

Nil.

Conflicts of interest

There are no conflicts of interest.

References

1. Grifith JF. "Problems in Urban air Pollution in AIAA". 8th ed. New York: Aerospace Science Meeting AIAA; 1970. p. 70-112.
2. Sharan M. Mathematical Aspects of Dispersion Modelling of Pollutants. New Delhi (INDIA): Indian Institute of Technology; 1994. p. 207-16.
3. Pasquill F, Smith FB. Atmospheric Diffusion. 3rd ed. Chichester, England: Ellis Horwood Ltd.; 1983. p. 437.
4. Dilley JF, Yen KT. Effect of a mesoscale type wind on the pollutant distribution from a line source. Atmos Environ 1971;5:843-51.
5. Arora U, Gakkhar S, Gupta RS. Removal model suitable for air pollutants emitted from an elevated source. Appl Math Model 1991;15:386-9.
6. Ermak DL. An analytical model for air pollutant transport and deposition from a point source. Atmos Environ 1977;11:231-7.
7. Pramod K, Maithili S. An analytical model for dispersion of pollutants from a continuous source in the atmospheric boundary layer. Proc R Soc A 2010;466:383-406.
8. Naresh R, Nath R. The effects of chemical reactions on the dispersion of air pollutants over area sources. Earth Moon Planet 1990;48:99-111.
9. Joseph V, Agarwal M. Mathematical modelling on the effect of chemical reaction on the air pollutant under mesoscale wind. GANITA 2020;70:73-84.
10. Buske D. An analytical solution of the advection-diffusion equation considering non-local turbulence closure. Environ Fluid Mech 2007;7:43-54.
11. Wortmann S, Vilhena MT, Moreira DM, Buske D. A new analytical approach to simulate the pollutant dispersion in the PBL. Atmos Environ 2005;39:2171-8.
12. Pal D, Khan S. A time dependent mathematical model for dispersion of air pollutants from point sources. Int J Environ Stud 1990;35:197-208.
13. Khan SK, Venkatachalappa M, Pal D. Three-dimensional analytical model of atmospheric dispersion of pollutant in a stable boundary layer. Int J Environ Stud 1992;41:1-2, 133-149.
14. Narayanachari KL, Suresha CM, Prasad MS, Pandurangappa C. Time dependent advection diffusion model of air pollutants with removal mechanisms. Int J Environ Sci 2014;5:34-50.
15. Lakshminarayanachari K, Suresha CM, Siddalinga Prasad M, Pandurangappa C. Numerical. Int J Res Environ Sci Technol 2013;3:9-18.
16. Arden PC, Dockery DW, Schwartz J. Review of epidemiological evidence of health effects of particulate air pollution. Inhal Toxicol 1995;7:1-18.
17. Lakshminarayanachari K, Sudheer Pai L, Siddalinga Prasad M, Pandurangappa C. A two dimensional numerical model of primary pollutant emitted from an urban area source with mesoscale wind, dry deposition and chemical reaction. Atmos Poll Res 2013:4:106-16.
18. Verma VS, Srivastava U, Bhandari PS. A mathematical model on dispersion of air pollutants. Int J Sci Res 2015;4:1904-7.
19. Nirmaladevi PV, Lakshminarayanachari K, Pandurangappa C. Three dimensional analytical model for the dispersion of air pollutants emitted from elevated point source with mesoscale wind and removal mechanism. J Comput Theor Nanosci 2018;15:3287-90.